Power-law Tradeoffs Between Optical and Electronic Switching

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***Abstract*—Designing a transport network requires finding the most cost-effective combination of electronic and optical switching to support the given demands. This task is not trivial because the solution space is so large. Furthermore, there are many interacting design variables. In electronic switching, the cost is a function of the number of packets that are switched in the routers, which directly corresponds to the average hop count ($\bar{H}$). The dominant cost of optical switching is a function of the total number of wavelengths used in the network (S) or the maximum number of wavelengths used on any fiber (W) and the number of transceivers (L) used. In this paper, we show that there is a power-law relationship between each pair of these variables, and we quantify the power-law exponent for many different types of topologies. Our results allow a network designer to choose not only the most cost-effective combination of electronic and optical switching, but also the best fiber topology to achieve the minimal total cost.**

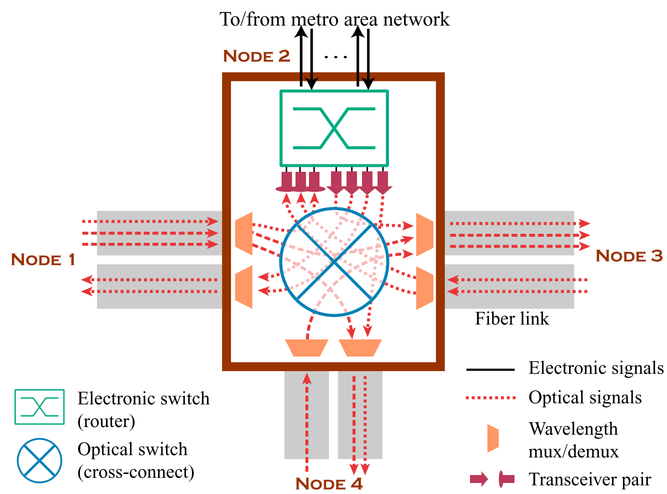

Fig. 1. Detail of node equipment and optical & electronic signals.

I. Introduction

It is widely recognized that fiber optic communications is and will continue to be an essential part of the Internet infrastructure, from access networks to long-haul networks. There are no better means to support the high volume of traffic demanded by today's applications, over sufficiently long distances.

Digital audio and video are already in widespread use in both business and personal arenas, and users are no longer confined by geographical locality in the multimedia content they request. Such content is now routinely downloaded through the Internet from geographically remote sites for playback on personal computers and portable players, from content distribution stores or via peer-to-peer networking. These are large aggregate demands over long distances.

The type of network considered in this work is a backbone transport network which connects multiple cities together with fiber links, employing dense wavelength-division multiplexing (DWDM). The job of the transport network is to carry traffic between the various cities. Each city corresponds to a node, or Point-of-Presence (POP). Such a node is shown in Fig. 1. It connects the transport network to the metropolitan area network serving the city, i.e., various local clients in the metropolitan area around a city send and receive traffic to/from other cities via the POP. Because the node is part of the transport network, it also handles transit traffic passing through the node on its way to its destination, e.g., from Node 1 to Node 3 in the figure. (A POP may also need to serve the local traffic among the various clients or service providers connected at a city, i.e., data which never enter the transport network. These demands are outside the scope of our study, and therefore self-traffic is always considered to be zero.)

The next-generation Internet backbone is likely to employ both optical and electronic switching, not only to reduce the total cost, but also to balance the advantages and disadvantages of each technology, as we shall explain.

Optical switching is a widely-accepted answer to the problem of having large demands which need to be carried over long distances. An optical cross-connect (OXC), or optical switch, (made possible by advances in MEMS, thermo-optics, and liquid-crystal technologies), allows the establishment of *lightpaths* [1], end-to-end all-optical channels, between pairs of nodes. An incoming optical signal at a node is routed to the appropriate outgoing fiber without termination (thus no transceivers involved) or electronic processing. In addition, as indicated in Fig. 1, the optical switch routes the lightpaths originating or terminating at the node between the transceivers and the appropriate fiber links.

There are many advantages of employing optical switching. It reduces the cost of transporting large quantities of data, not only by eliminating costly opto-electronic conversion, but also by avoiding per-packet inspection. Because no buffer is used in optical switching, there is no queueing delay. Furthermore, optical switching is agnostic to the underlying data rate of an optical channel (up to a certain point, as the channel bandwidth limits the maximum data rate), and thus, the OXCs do not have to be upgraded when the interface speed is increased.

Electronic switching, on the other hand, also offers several benefits. With electronic switching, data packets or flows can be processed and aggregated at a much finer granularity compared to the large bandwidth of a lightpath, which is typically greater than that required between two nodes. Generally, electronic switching increases the capacity utilization, from both aggregation and statistical multiplexing, and reduces the wavelength requirements, in terms of both the wavelength multiplexing density and the total number of optical channels.

To handle local traffic, i.e., traffic to/from the metropolitan area network, a node must properly sort and multiplex the traffic to go on the various optical signals originating at or terminating at a node. We consider this functionality, *local-interface switching*, to be part of the transport network (i.e., in our scope of study), and it requires electronic processing. Furthermore, the node may perform electronic processing of transit traffic—data not destined to it. Every node has an electronic switch to handle the local-interface switching and the switching of transit traffic, as indicated in Fig. 1.

No discussion of switching cost is complete without including the cost of transceivers. When an optical signal, or lightpath, is terminated at a node, it is converted to/from an electronic signal. The termination equipment is a transceiver, i.e., every lightpath established requires one pair of transceivers. Now, consider a demand, which may take one or more lightpaths from source to destination. Transceivers are needed at both the source and destination nodes. At intermediate nodes, transceivers are needed at every electronic switch interface, but not needed when the signal is optically switched. In some cases, the total number of transceivers may be reduced due to optical bypass, while in other cases, transceivers may be reduced due to the aggregation advantages of using electronic switching.

In the design and dimensioning of a network, it is very useful to be able to capture simply how the costs of optical switches, electronic switches, and transceivers relate to each other, as well as to the fiber topology. Choosing the right combination is very hard without an understanding of the quantitative relationships among optical switching, electronic switching, transceiver requirements, and topology, since the number of possible solutions is enormous. But when armed with an understanding of the fundamental tradeoffs, solutions to fundamental design issues can be roughly but quickly determined. It is important to understand the entire spectrum between the electronic switching extreme (lightpaths connecting only physical neighbors) and the optical switching extreme (all node pairs are connected by dedicated lightpaths). In this paper, for the first time, these relationships are quantified.

A. Contributions of this paper

The primary focus of this paper is to quantify the relationships among optical switching, electronic switching, transceiver requirements, and topology. We show that power-law relationships exist for all topologies that we analyze, and we quantify the value of the exponent for various topologies. Based on these tradeoffs, the lowest-cost network design is determined.

Employing several methods of analysis, we characterize the relationships among three measures of cost—the wavelength requirement (corresponding to optical switching and transmission costs), the number of lightpaths (corresponding to transceiver costs), and the number of lightpath hops (corresponding to electronic switching and performance costs). We find that the distribution of the physical lengths of the lightpaths is the dominant factor influencing these relationships.

The interaction between optical switching and electronic switching has been studied before. In [2], the authors studied the fractional reduction in router size when optical switching is added. However, they did not consider how optical switching trades off with electronic switching, the focus of this paper.

B. Outline of paper

In the next section, we describe the models, assumptions, and approach we use. We also explain the three methods we follow to analyze different aspects of the problem.

Then, we examine the tradeoffs among the cost measures for two different types of topologies—rings in Sec. III and short-fiber topologies in Sec. IV. We have also analyzed other topologies, such as stars, unbiased random topologies and biased random topologies using the Waxman model [3]. We have shown that the tradeoffs in these topologies also follow a power-law. Because of space limitations, we refer interested readers to the technical report [4].

These power-law tradeoffs are useful for the design of networks, and in Sec. V we show how the entire switching tradeoff can be determined by three topological parameters: the number of nodes, the number of edges and the average physical distance between node pairs; we also show how to find the optimal fiber layout and the optimal switching hardware configuration.

In Sec. VI, we explore the case when not all nodes may be equipped to perform electronic switching of transit traffic. Lastly, Sec. VII deals with generalized traffic patterns: how the tradeoffs are affected by the magnitude of the traffic and by non-uniform traffic.

II. Models, Approach, and Assumptions

A. Architectural model and key cost measures

We consider WDM optical networks in which nodes are capable of both optical and electronic switching. There are no restrictions on the presence of or amount of equipment (transceivers, optical cross-connects, electronic routers) used at any node, i.e., they are available should we choose to employ them. Furthermore, we are free to choose the optical routes (the fiber links taken by lightpaths) without regard to the availability of a particular wavelength (color); regardless, we have found nearly identical results with and without the wavelength continuity constraint.

A traffic demand from one node to another, also called a flow, is handled by either an optical switch or an electronic switch at each intermediate node along the path. We use the term *physical hops* to refer to the number of fiber links a lightpath or a packet passes through. The *lightpath length* is expressed in terms of the number of physical hops. We use the term *logical hops* to refer to the number of lightpaths a packet in a flow traverses, also known as the hop count. We assume that the traffic always takes the shortest path in terms of the number of logical hops.

Throughout this paper, a lightpath refers to a symmetric pair of transparent optical channels, and L represents the total number of lightpaths in the system. That is, if there is an optical channel from node A to node B, there will be an optical channel from node B to node A, routed along the same physical path, constituting one lightpath. We also assume that they are established on the same wavelength (color).

Optical switching cost: There exists at every node an optical cross-connect that has no connectivity or size constraints. The amount of optical switching performed at a node depends on the number of lightpaths the switch needs to handle, and we consider that the cost is proportional to the total number of optical switching occurrences σ, defined as the sum over all lightpaths of the number of times each lightpath is switched. If lightpath l traverses x_l physical hops, it is switched $x_l - 1$ times at intermediate nodes, and 2 times at the terminal nodes, once at each end. The total optical switching is

$$\sigma = \sum_{l=1}^{L} (x_l + 1).$$

In our analysis, we define $S = \sum_{l=1}^{L} x_l$, equivalently the sum over all fiber links of the number of utilized wavelengths on each link. Thus, the total optical switching $\sigma = S + L$. Although cost is measured by σ, we characterize the tradeoffs using S and L.

The second measure of the optical requirement is W, the maximum number of wavelengths used on any fiber link in the entire network. W (along with the nodal degree which is fixed for a given topology) determines the size of the OXC used at every node. The bigger the size, the more costly the OXC is likely to be. W also determines the channel spacing between neighboring wavelengths. A smaller W allows coarser channel spacing; as a result, less expensive lasers could be used to originate lightpaths. Conversely, a large W forces one to use denser channel spacing in order to fit all wavelengths in the same fiber. Therefore, costly lasers such as those with little thermal drift have to be used.

Electronic switching cost: We measure electronic switching by the total volume of switched traffic, i.e., the number of times each flow is switched electronically, weighted by the magnitude of the flow. As discussed in the introduction, the electronic switching at a node is composed of the local-interface switching and the transit-traffic switching. Every flow undergoes local-interface switching once upon entering the network and once when it arrives at the destination node. In addition, a flow undergoes transit-traffic switching every time it is electronically switched at an intermediate node. The total number of times a flow incurs electronic switching is the number of logical hops + 1.

Thus, the electronic switching cost is proportional to the average number of times a flow is switched, $\bar{H} + 1$, where $\bar{H}$ is the average logical hops taken by all demands.

Transceiver cost: The number of transceivers is an important contributor to the total cost, as the lasers used in the transceivers tend to be costly components. The total number of lightpaths in the system, L, corresponds to the number of transceivers; thus, the cost of the transceivers is proportional to L.

B. Traffic model

To facilitate discussion and analysis, in Sections III and IV, we consider uniform all-to-all traffic and assume that the lightpath capacity is large enough to accommodate any traffic demands that need to be multiplexed onto a lightpath, such that all demands can be routed along the shortest path. This isolates the tradeoff behavior from the effect of a particular traffic matrix. In Sec. VII, we examine the effect of having a bandwidth constraint on the lightpaths and the effect of having nonuniform demands.

C. Approach to solution

Our goal is to explore the entire region of the tradeoff, from the electronic switching extreme to the optical switching extreme. The basic approach to finding the tradeoff is simple: vary, from one extreme to the other, the available resources corresponding to optical switching cost (S and/or W), and find at each point the design having the minimum electronic switching cost (directly proportional to $\bar{H} + 1$). This is the same objective as that in [5] and is a variant of the logical topology design problem [6] [7] [8] [9]. That is, for a given physical topology, and for each given value of S (or W), find the logical topology that minimizes the average logical hop count $\bar{H}$, while satisfying demands between all pairs of nodes.

Design of the logical topology: Our approach to finding the logical topology—the set of lightpaths, specified by their sources and destinations—is to maximize the total number of lightpaths set up, L, as doing so maximizes the number of one-hop connections and minimizes the number of node pairs requiring multiple logical hops. Since in general, W (or S) is not sufficient to allow the establishment of a lightpath between every pair of nodes, the way to maximize L is to set up shorter lightpaths before setting up ones requiring more physical hops (i.e., more resources). First, at the electronic switching extreme, only neighbors are connected with lightpaths. As more wavelengths become available, longer and longer lightpaths may then be established. Besides producing good results, this approach has the added advantage of allowing mathematical analysis in a number of regular topologies.

Unifying variable: Above (Sec. II-A) we identified several key cost metrics: the number of wavelengths W (or S), the average number of logical hops $\bar{H}$, and the total number of lightpaths L. In order to determine their mathematical relationships, for instance, to write $\bar{H}$ in terms of L, we relate each of the three metrics W, L, and $\bar{H}$ to a common variable, t.

The variable t represents the longest lightpath length among all lightpaths in the logical topology. In other words, it is the number of physical hops to the most distant node reachable via one lightpath. This is a measure which can be used in all scenarios, because the shorter lightpaths are established before the longer ones. A given W determines that lightpaths up to a certain length t can be established; conversely, given t, we can find how many wavelengths W are required. Given t, we also know exactly which lightpaths may be established, thus yielding L. Since the logical topology is known, $\bar{H}$ can also be determined.

As we shall see, there are pairwise power-law relationships among W (or S), L, $\bar{H}$, and t. This means that the relationship between any two of these variables x and y can be written as $y = ax^e$, where a is a constant and e is the exponent of the power law. We use the notation $y(x)$ to denote the exponent relating y to x, i.e., $y(x) = e$.

Lightpath length distribution: Our work reveals that the lightpath length distribution (LLD) greatly influences the tradeoff. The LLD is described by n_i, which denotes the number of node pairs that are i physical hops apart along the least-hops path (the physical route taken by a lightpath connecting the node pair).

The LLD is important to the tradeoff because it affects which, and how many, lightpaths can be supported for a given limit on W (or S). At the optical switching extreme, lightpaths are established between all pairs, but as W (or S) is reduced, the longest lightpaths are the first to be removed. Therefore, for each value of W (or S), the number of lightpaths is known, their lengths ("reach") are known, and the average hop count is determined.

Because of its importance, we shall pay particular attention to the influence of the LLD when examining the various topologies.

D. The particular methods employed

We employ three particular methods in our study, all using the same design principles. No single method is best on all accounts, but judiciously employed and properly analyzed, these three methods provide a complete picture to explain the tradeoff relationships. We hereby describe these methods and when they are used:

1) The analytical method: For several regular topologies, we can derive closed-form expressions relating W (or S), L and $\bar{H}$ and, therefore, directly express their tradeoffs. This method lends insight into the behavior of the curves we observe.

When deriving the equations, especially for $\bar{H}$, a ceiling or flooring operator may be needed in order to be mathematically precise. However, for simplicity, we ignore such operators in our derivations. This approximation is quite crude and can result in a noticeable difference from the real behavior. We also make other approximations, e.g., ignoring lower-order terms, in order to make the analysis tractable. These approximations are more accurate, producing results very similar to the real behavior. Although taking these necessary approximations allows the trends to be highlighted, some details of the actual behavior are obscured, leading us to also employ the next method.

2) The semi-analytical method: In this method, employed when the LLD is known, W (or S), L and $\bar{H}$ values are numerically computed for each value of t, based on the LLD.

For a given value of t, S is found by summing all lightpaths up to t physical hops: $S = \sum_{i=1}^{t} i \cdot n_i$. Since this sum has to be less than MW (M is the number of fiber links in the physical topology), we can also determine the lower bound on W. The corresponding number of lightpaths established is $L = \sum_{i=1}^{t} n_i$. We can also calculate the average hop count $\bar{H}$ by assuming that a flow that goes through i physical hops will go through $\lceil \frac{i}{t} \rceil$ logical hops, since the longest lightpaths are taken to reach the destination in minimal hops. Once we calculate the S (and the lower bound on W), L, and $\bar{H}$ values for each t, we can plot them on a log-log scale and use the least squares method to find the exponents, thus deriving all pairwise relationships.

Both the analytical and semi-analytical methods allow us to evaluate very large topologies. However, neither can take into account the actual packing of wavelengths, except for very simple topologies in which the wavelength routing is known. Generally, the packing of wavelengths becomes progressively more difficult as longer lightpaths are added, and it is possible that a particular wavelength color is not used on most fiber links. This leads to a higher value of W as compared to the lower bound given by $\frac{S}{M}$. To see this effect, actual simulation of the scenario must be carried out. Simulation is the method showing the most realistic results, but even with an efficient heuristic like ours, it is limited in the number of cases and the number of nodes that can be handled.

3) The simulation method: The simulation method is only used when W (rather than S) is the metric of interest. The problem of minimizing $\bar{H}$ subject to W wavelengths can be readily formulated as an Integer Linear Programming (ILP) problem [5]. However, solving the ILP directly is very time consuming. Therefore, heuristic algorithms are needed. The work in [5] considered the same objective as ours, but the two heuristic algorithms proposed assume there is a limit on the number of transceivers and also that the traffic matrix is not uniform, so that the structure of the traffic matrix could be exploited. In many cases, the algorithms will route longer lightpaths before the shorter ones, which is very different from our approach of first routing the shorter lightpaths.

We propose and employ a heuristic algorithm, called *Shortest Lightpaths First* (SLF). As in all three methods, establishing shortest lightpaths first is the strategy used to maximize the total number of lightpaths established, L. The algorithm takes as input the given fiber topology and W.

The SLF heuristic algorithm has four phases.

- Route all $\frac{N(N-1)}{2}$ lightpaths using the shortest path. Note that at this point, some links may be *overloaded*, meaning that more than W lightpaths are routed over it.
- Examine each lightpath in turn, in decreasing order of physical hop length. If every fiber link on its shortest path is overloaded, remove the lightpath.
- Rank all routed lightpaths based on the number of overloaded links on their shortest path. The one using the most number of overloaded links is removed. Repeat this phase until no link is overloaded. At the end of this phase, the lightpaths which remain routed can be feasibly

established using W wavelengths.

- In order of increasing physical hop length, attempt to add each of the removed lightpaths over alternate paths (considering the unused wavelengths).

Employing this simulation method, we collect L and $\bar{H}$ data for various values of W, then use least squares curve fitting to determine the power-law tradeoff.

E. Solution approach comparison

Throughout this paper, we focus on quantifying the tradeoff under the shortest lightpaths first approach. We have done extensive comparison with other solution approaches, such as the one in [5], where node pairs that are farthest apart in the current logical topology are connected first by a direct lightpath. We note that our approach performs better in many cases, especially when W (or S) is large. When W (or S) is smaller, the difference in $\bar{H}$ is small enough such that it will not affect our power-law observation. The comparison between the two different approaches is shown in Fig. 2 for a ring topology with 50 nodes. We also compared results from our solution approach with the optimal solutions from ILP formulations for many problem instances and found that our approach produces near-optimal results.

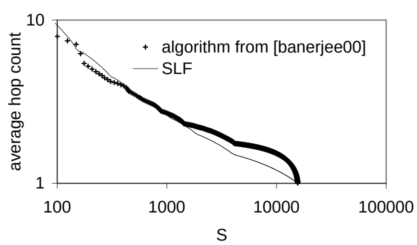

Fig. 2. Comparison with the solution approach in [5] in a ring topology with 50 nodes.

III. Tradeoffs in rings

In this section, we study tradeoffs for the ring topology. We employ the analytical and semi-analytical methods and compare their results. Due to the well-defined topological structure, simulation is not needed.

We consider a ring physical topology in which the nodes are numbered from 0 to $N-1$ in the clockwise direction. Traffic and lightpaths on the ring can be routed in either direction, and the shorter path is always taken. For simplicity of discussion, we analyze rings with an even number of nodes.

In a ring topology, the lightpaths range in length from 1 to $N/2$ physical hops. It takes exactly one wavelength to establish all lightpaths that are one physical hop long. This one wavelength is reused on every fiber link to connect the two neighboring nodes. Similarly, it takes two wavelengths to establish all lightpaths that are two physical hops long. In general, to route all lightpaths that are i physical hops long, i wavelengths are needed.

As discussed above, our method sets up shorter lightpaths first. Let us assume that lightpaths up to t physical hops long can be established. Then we have

$$W = 1 + 2 + \ldots + t = \frac{t(t+1)}{2} \approx \frac{t^2}{2}$$

Because of symmetry, S is simply NW.

There are N lightpaths that are i physical hops long for all i except $i = \frac{N}{2}$. When $i = \frac{N}{2}$, there are only $\frac{N}{2}$ lightpaths. For simplicity of analysis, we will ignore this and assume that $n_i = N \; \forall i$. Summing up all lightpaths that are up to t physical hops long, we have

$$L = N \cdot t$$

In determining the average hop count $\bar{H}$, because of symmetry, it is sufficient to consider only the lightpaths that originate from a single node. Without loss of generality, let us consider all lightpaths from node 0. Since we have established all lightpaths up to t physical hops long, the traffic from node 0 to node i will take $\lceil \frac{i}{t} \rceil$ logical hops. Similarly, the traffic from node 0 to node $N/2+1-i$ will take $\lceil \frac{N/2+1-i}{t} \rceil$ logical hops. Ignoring the ceiling operators, the average of the two is $\frac{N/2+1}{2t}$. Since this is true for all i, we have the following relationship, which shows that $\bar{H}$ scales inverse-linearly as t:

$$\bar{H} = \frac{N/2+1}{2t} \approx \frac{N}{4t} \quad (1)$$

Having derived the expressions for W, S, L, $\bar{H}$ as a function of t, we can now derive pairwise relationships between all variables. In particular, $L(W) = L(S) = 0.5$ and $\bar{H}(W) = \bar{H}(S) = -0.5$, i.e., for a constant increase in L and, hence, a constant decrease in $\bar{H}$, the wavelength requirement (W and S) increases much faster. This is because we establish longer lightpaths, which require more wavelengths, after we establish shorter ones first.

The approximation used in equation (1) (ignoring the ceiling operators) is very crude. To illustrate what really happens, let $t_m = \frac{N}{2}$ denote the length of the longest lightpath. If $t = \frac{t_m}{2}$, node 0 can at most reach the $\frac{N}{4}$-apart neighbor, therefore, half of the traffic from node 0 will go through one hop and the other half will go through two hops. The average hop count $\bar{H} = 1.5$. Similarly, when $t = \frac{t_m}{4}$, $\bar{H} = 2.5$. When $t = \frac{t_m}{8}$, $\bar{H} = 4.5$ and so on. This is shown in Table I.

TABLE I

$\bar{H}$ AS A FUNCTION OF t.

t	$\frac{t_m}{2}$	$\frac{t_m}{4}$	$\frac{t_m}{8}$	$\frac{t_m}{16}$	$\frac{t_m}{32}$	$\frac{t_m}{64}$	...
$\bar{H}$	1.5	2.5	4.5	8.5	16.5	32.5	...

Every time t is reduced by half, $\bar{H}$ does not double. Therefore, $\bar{H}$ does not exactly scale inverse-linearly as t. Using the semi-analytical method, we find that $\bar{H}(t) = -0.8$ and $\bar{H}(W) = -0.4$.

In Fig. 3, we show the tradeoff between W and $\bar{H}$ for various values of N, as found using the semi-analytical method, along with the corresponding fitted curve.

Even though $\bar{H}$ is not exactly a linear function of W in the log-log plot as shown in the figure, we find that the correlation coefficient to a straight line is more than 99%, suggesting that a power-law is a good fit. The non-linear shape of $\bar{H}$ is an effect of the ceiling operator. Although not shown, we note that the L curve as a function of W is almost perfectly straight on a log-log plot.

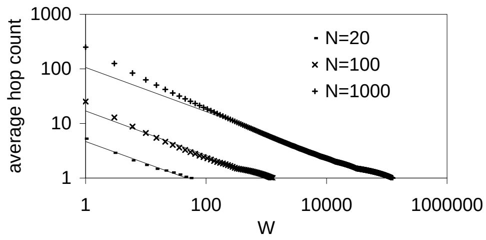

Fig. 3. $\bar{H}$ as a function of W in the ring topology, found using the semi-analytical method.

IV. TRADEOFFS IN SHORT-FIBER TOPOLOGIES

In this section, we consider a particular type of mesh topology called the short-fiber topology. Many real life topologies could be classified as short-fiber topologies. A common characteristic of these topologies is that fiber links tend to be short, connecting physically neighboring nodes. In doing so, the diameter of the network tends to be larger, and there are many long lightpaths. Short-fiber topologies can also be characterized by a small expansion factor, i.e., fewer new nodes are reached when we connect more and more longer lightpaths.

In a short-fiber topology, the maximum hop count is on the order of $\sqrt{N}$ at the electronic switching extreme; thus, it is considered a two-dimensional topology. Compared to the ring (one-dimensional), there is a large constant reduction in $\bar{H}$. On top of that, we will see that the power-law exponent is also smaller.

To understand the tradeoffs in short-fiber topologies, we first look at two special instances—the torus and regular mesh topology. Their regular structures allow us to apply the analytical and semi-analytical methods. Then, we move to arbitrary short-fiber topologies.

A. Tori

We first consider the Torus topology, as shown in Fig. 4. For simplicity of presentation, we assume there are $2k+1$ nodes on each side of the torus, and $(2k+1)^2$ nodes in the torus altogether. Nodes are symmetric to each other. Therefore, it is sufficient to only consider lightpaths originating from a single node, e.g., the center black node in Fig. 4.

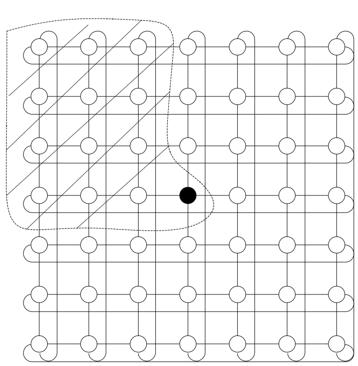

Fig. 4. A torus network with 7x7 nodes. $k = 3$.

To derive the lightpath length distribution (LLD), we can just focus on the upper-left quadrant and multiply the results by $\frac{4N}{2}$ because of symmetry. The $\frac{4N}{2}$ factor comes from the fact that there are 4 quadrants, that we have to sum up lightpaths originating from all N nodes, and that the bidirectional lightpaths have been double counted. From the highlighted quadrant in the figure, we see one node 1 hop away, two nodes 2 hops away, ..., ending with one node 6 hops away. Thus, $n_i = 2Ni$ for $i \leq k$, and $n_i = 2N(2k+1-i)$ for $i > k$.

Again because of symmetry, lightpaths of length i from all nodes need exactly i wavelengths. Let us assume lightpaths up to t physical hops long can be established and $t \leq k$. The number of wavelengths used by lightpaths up to t physical hops long is

$$W = \sum_{i=1}^{t} i \cdot i = \frac{1}{3}t^3 + \frac{1}{2}t^2 + \frac{1}{6}t \approx \frac{1}{3}t^3 \qquad (2)$$

Note that, because of symmetry, S is exactly $2NW$.

We can derive the number of lightpaths established as follows

$$L = \frac{4N}{2}\sum_{i=1}^{t} i \approx Nt^2 \qquad (3)$$

To derive the average hop count, we can apply the averaging argument that we used for rings. We note that flows that are i physical hops long will go through $\lceil \frac{i}{t} \rceil$ logical hops, and flows that are $2k+1-i$ physical hops long will go through $\lceil \frac{2k+1-i}{t} \rceil$ logical hops. Ignoring the ceiling operators, the average of the two is $\frac{2k+1}{2t}$. Since this is true for all i, we have

$$\bar{H} = \frac{2k+1}{2t} \approx \frac{k}{t} \qquad (4)$$

Having derived the expressions for W, S, L, $\bar{H}$ as a function of t, we can now derive pairwise relationship between all variables. $W(t) = S(t) = 3$ and $L(t) = 2$, therefore, $L(W) = L(S) = \frac{2}{3}$. Also, $\bar{H}(t) = -1$, therefore, $\bar{H}(W) = \bar{H}(S) = -\frac{1}{3}$.

We can similarly derive W(or S), L and $\bar{H}$ for the case of $t > k$. The full derivation is shown in the technical report [4]. When t is just a little larger than k, W (and S) and L still scale as t^3 and t^2, respectively. However, when t gets closer to $2k$, W (and S) and L get further away from t^3 and t^2, respectively.

In Fig. 5, we show L and W as a function of t for $k = 5$ and $k = 50$. For the most part, both L and W are straight lines in the log-log plot. However, when t approaches $2k$, both L and W curve down and the slope of L seems to flatten proportionally to the slope of W. As a result, when we plot L as a function of W on a log-log scale, it appears to be a straight line with a slope of $\frac{2}{3}$, i.e., $L(W) = \frac{2}{3}$.

If we plot $\bar{H}$ as a function of t using the semi-analytical method, the shape of the curve is very similar to that in Fig. 3, suggesting that equation (4) made the same approximation as we have seen for rings. However, if we plot $\bar{H}$ as a function of W, as shown in Fig. 6, the shape is slightly different. In particular, $\bar{H}$ curves up when W is large. This is because W does not grow as t^3 any more when t is large, as shown in Fig. 5.

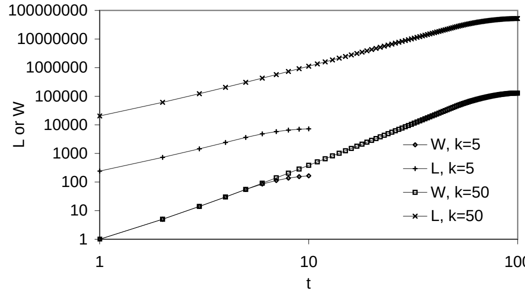

Fig. 5. L and W as a function of t in the torus topology for $k = 5$ and $k = 50$.

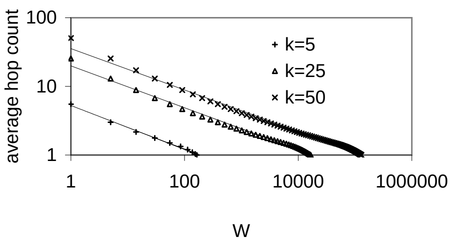

Fig. 6. $\bar{H}$ as a function of W in the torus topology for $k = 5$, $k = 25$ and $k = 50$.

In Table II, we show the exponents from curve fitting using the semi-analytical method for several values of k. All curve fittings have a correlation coefficient of more than 0.994.

TABLE II

EXPONENTS FROM CURVE FITTING USING THE SEMI-ANALYTICAL METHOD IN THE TORUS TOPOLOGY.

k	5	50	500
$W(t)$, $S(t)$	2.31	2.69	2.80
$L(t)$	1.54	1.79	1.86
$\bar{H}(t)$	-0.74	-0.81	-0.83
$L(W)$, $L(S)$	0.67	0.66	0.66
$\bar{H}(W)$, $\bar{H}(S)$	-0.32	-0.30	-0.29
$\bar{H}(L)$	-0.48	-0.45	-0.44

As shown, $W(t)$ is not 3 and $L(t)$ is not 2 as suggested by equations (2) and (3), although they do approach 3 and 2 respectively when N is large. This suggests that the flattening of W and L in Fig. 5 has less effect when N is large. $L(W)$ stays roughly constant at $\frac{2}{3}$, suggesting that L and W indeed flatten at the same rate. $\bar{H}(t)$ is roughly -0.8, as we have seen for rings. $W(t)$, $L(t)$ and $\bar{H}(t)$ together determine the values of $\bar{H}(W)$ and $\bar{H}(L)$. In particular, $\bar{H}(W)$ is very close to $\frac{1}{3}$, as suggested by our analysis, and its absolute value decreases slightly when N is large, mostly because $W(t)$ increases.

In tori and in short-fiber topologies in general, $\bar{H}(W)$ is close to $\frac{1}{3}$ and is not significantly affected by N. However, this is not true for random topologies, where the absolute value of $\bar{H}(W)$ decreases as N gets larger. For details, please refer to the technical report [4].

B. *Regular meshes*

Moving one step closer to arbitrary short-fiber topologies, we now consider the regular mesh topology, i.e., the grid topology. Let m denote the number of nodes on each side ($N = m^2$). An example with $m = 4$ is shown in Fig. 7. Compared to the torus, there are more long lightpaths in the regular mesh because they can not wrap around as in the torus. Furthermore, because of the asymmetry between the edge and center nodes, the fiber links in the center use more wavelengths than the fiber links on the edge.

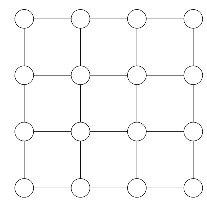

Fig. 7. A regular mesh topology with 4x4 nodes. $m = 4$.

We find the LLD as follows. We number the nodes on a Cartesian coordinate system from (0,0) to ($m-1$,$m-1$). With shortest path routing, the length of a lightpath between a pair of nodes (p,q) and (r,s) is the number of hops taken in the Y direction plus the hops in the X direction: $|r-p|+|s-q|$. Thus, the number of lightpaths of length i is expressed as

$$n_i = \sum_{p=0}^{m-1}\sum_{q=0}^{m-1}\sum_{r=0}^{m-1}\sum_{s=0}^{m-1} \frac{I_i\left(|r-p|+|s-q|\right)}{2}$$

where $I_i(|r-p|+|s-q|)$ is the indicator function with value 1 if $i = |r-p|+|s-q|$, and 0 otherwise. A closed-form expression for n_i has also been derived; the details are in [4].

In Fig. 8, we show the LLD (n_i) for different values of m as derived from the equations above. All curves follow the same shape. In fact, this is also true for larger regular meshes. The shape is close to that of the Torus topology, where n_i increases close to linearly and then decreases close to linearly. However, there are two differences. First, the top is more rounded, which will cause S and L to grow slower. Second, the curve is no longer symmetric, which means that $\bar{H}(t)$ will differ from that of the torus topology because the averaging argument no longer holds.

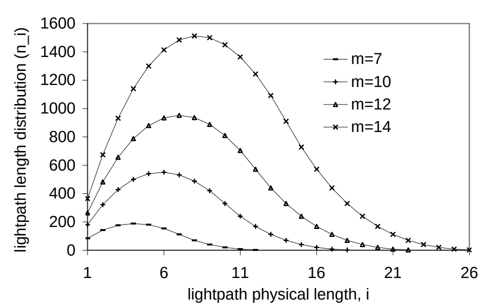

Fig. 8. LLD in the regular mesh topology for different values of m.

Because of the similarity in LLD, the tradeoffs are very close to that of the torus topology. In Fig. 9, we plot L and

S as a function of t using the semi-analytical method. The shape is very similar to that in Fig. 5. However, both S and L curve downwards more when t is big compared to the torus topology. In Fig. 10, we plot $\bar{H}$ as a function of S. Again, the shape is very close to that in Fig. 6.

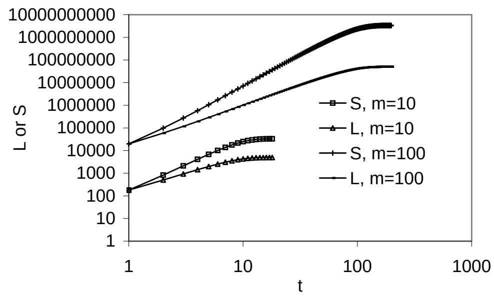

Fig. 9. L and S as a function of t in the regular mesh topology for $m = 10$ and $m = 100$.

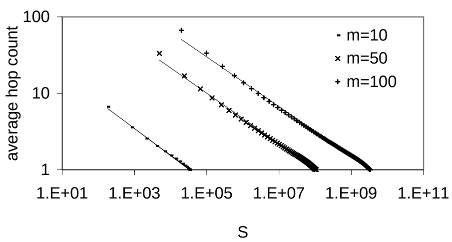

Fig. 10. $\bar{H}$ as a function of S in the regular mesh topology for $m = 10$, $m = 50$ and $m = 100$.

In Table III, we show the exponents from curve fitting using the semi-analytical method for several values of m. All curve fittings have a correlation coefficient of more than 0.997. Compared to the torus, $S(t)$ and $L(t)$ are further away from 3 and 2 respectively because of the rounded LLD, which causes S and L to flatten more when t is large, as shown in Fig. 9. As in tori, $S(t)$ and $L(t)$ increase slowly when N increases. $S(t)$ and $L(t)$ increase at roughly the same rate because $L(S)$ stays around 0.63. $\bar{H}(t)$ ranges from -0.62 to -0.73, smaller than that in the torus topology. This is mostly because of the asymmetrical LLD. $\bar{H}(S)$ decreases from -0.36 to -0.32 as N increases, and the exponents are higher than those in the torus topology.

TABLE III

EXPONENTS FROM CURVE FITTING USING THE SEMI-ANALYTICAL METHOD IN THE REGULAR MESH TOPOLOGY.

m	7	10	12	14	100	1000
$S(t)$	1.71	1.82	1.87	1.91	2.19	2.28
$L(t)$	1.07	1.14	1.17	1.20	1.39	1.44
$\bar{H}(t)$	-0.62	-0.64	-0.65	-0.66	-0.71	-0.73
$L(S)$	0.63	0.63	0.63	0.63	0.64	0.64
$\bar{H}(S)$	-0.36	-0.35	-0.35	-0.35	-0.33	-0.32
$\bar{H}(L)$	-0.58	-0.56	-0.55	-0.55	-0.51	-0.50

If W is the metric of interest, we have to employ the simulation method to accurately account for the uneven wavelength usage of different fiber links. In Table IV, we show the exponents from curve fitting using the simulation method for several values of m. In addition to W, L and $\bar{H}$, we also show the utilization u, which is defined as $\frac{S}{MW}$. Compared to the semi-analytical method (Table III), both $L(W)$ and $\bar{H}(W)$ are smaller than $L(S)$ and $\bar{H}(S)$ respectively. There are two reasons that cause this difference. The first is the reduced wavelength utilization as W increases. When W is large, many lightpaths, including long ones, are routed. These long lightpaths make it hard to fully utilize all wavelengths provisioned on each fiber link. Conversely, when W is small, only short lightpaths are routed. Therefore, the utilization could be greatly improved. For the cases we studied, the utilization ratios range from 80% or 90% when W is small to around 66% when W is large. Asymptotically, $u(W)$ is around -0.15 as shown in the table. The second reason is that, because of the limit on W, some lightpaths are routed through non-shortest paths, which use more wavelength resources.

TABLE IV

EXPONENTS FROM CURVE FITTING USING THE SIMULATION METHOD IN THE REGULAR MESH TOPOLOGY.

m	7	10	12	14
$L(W)$	0.54	0.55	0.56	0.57
$\bar{H}(W)$	-0.27	-0.27	-0.26	-0.26
$\bar{H}(L)$	-0.51	-0.49	-0.47	-0.46
$u(W)$	-0.13	-0.15	-0.15	-0.15

C. *Arbitrary short-fiber topologies*

We now consider topologies that have been designed by a topological design algorithm under a fixed fiber-to-node ratio (roughly equivalent to the edge-to-node ratio if short fibers are used). The design algorithm is very conscious about fiber usage and tries to use shorter fibers as much as possible. The designed topologies are very similar to the regular mesh or the torus topology. In Fig. 11, we show the measured LLD for four generated topologies ($N = 50$, $N = 100$, $N = 150$ and $N = 200$) with a fiber-to-node ratio of 2 (average node degree of roughly 4). The graph is very similar to Fig. 8. The graphs will be similar if other values of the fiber-to-node ratio are chosen. Because of the similarity in LLD, the tradeoffs are very close to that of the regular mesh. In the interest of space, the detailed results are reported in [4].

D. *Several real topologies*

Many real-life topologies are short-fiber topologies. For example, Level 3 Networks' (an Internet backbone service provider) fiber topology, as shown in Fig. 12 [10], is clearly a short-fiber topology since many short fibers are used. In Table V, we report the power-law exponents for the NSF, ARPAnet and Level3 topologies. The numbers match very well with the results for the arbitrary short-fiber and the regular mesh topologies.

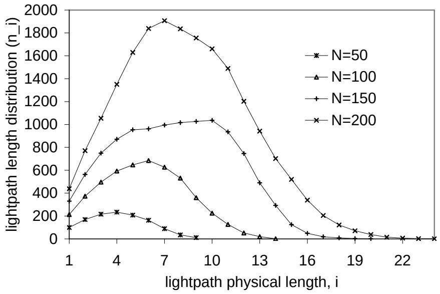

Fig. 11. LLD in the arbitrary short-fiber topologies for different values of N. The fiber-to-node ratio is 2 (node degree ≈ 4).

Fig. 12. Level3's backbone fiber topology.

V. Application to Network Design

In the previous sections, we have shown that the tradeoff between optical and electronic switching follows a power-law relationship. Understanding the tradeoff has very important applications in many network design scenarios. For example, if the power-law exponent is known, one can quickly determine how the cost will shift if a network design were to change such that different amounts of optical and electronic switching are employed. In this section, we will show another example of application to network design. We will show that, using the power-law information, the network designer can not only choose the best combination of optical and electronic switching, but can also choose the best fiber topology to balance the cost of fiber and switching.

A. Deriving the tradeoff relationship

Knowing that the tradeoff follows a power-law allows a network designer to quickly derive the tradeoff curve. One way is to derive two solution points and then connect a straight line through them in a log-log plot. For example, we can derive the two solution points from the two extremes of the tradeoff, one with the maximum amount of optical switching, one with the maximum amount of electronic switching.

TABLE V

Power-law exponents for several real network topologies.

Exponent	$L(W)$	$\bar{H}(W)$	$L(S)$	$\bar{H}(S)$
Level3	0.52	-0.29	0.60	-0.37
NSF	0.52	-0.28	0.65	-0.35
ARPAnet	0.56	-0.28	0.65	-0.34

If we use only one wavelength on each fiber link to connect the neighboring nodes ($W = 1$ and $S = M$), the logical topology is the same as the physical topology and maximum electronic switching will result. We denote the average logical hop count as $\bar{H}_0$ in this extreme case. If we connect all node pairs using a direct lightpath, the logical topology is a full mesh and minimal electronic switching will result ($\bar{H} = 1$). At this extreme, $S = \frac{N(N-1)}{2}\bar{H}_0$.

In Fig. 13, we find these two solution points for various topologies and connect a direct line between them. The ring and star topology have $N = 100$; the torus topology has $k = 5$; and the random topology has $N = 100$ and $M = 200$. The direct lines are in good agreement with our earlier analysis and simulations, suggesting that they are good approximations for the actual tradeoff. Note that $\bar{H}$ is small in the star topology when S is large. This is because all lightpaths in the star topology have the same length, and the smaller $\bar{H}$ comes at the cost of long fibers (details in [4]). We can similarly plot the tradeoff lines between L and $\bar{H}$, and they are also in very good agreement with our results.

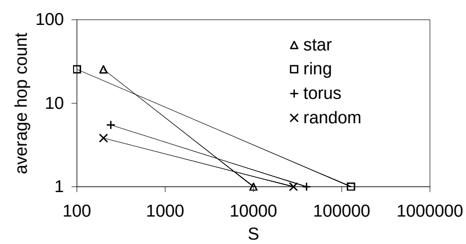

Fig. 13. The tradeoff curve obtained from two solution points in the log-log plot for different topologies.

If we know the value of $\bar{H}_0$, and since we already know the values of N and M, we can compute the slope of the line, i.e., the power-law exponent. Let us assume $\bar{H}_0 = aN^{\frac{1}{x}}$. Both a and x are constants, and x indicates the dimension of the topology. For an x-dimensional topology, $\bar{H}_0$ should be on the order of $O(N^{\frac{1}{x}})$. For example, in the ring, $\bar{H}_0 = \frac{1}{4}N^1$, and in the torus, $\bar{H}_0 = \frac{1}{2}N^{\frac{1}{2}}$.

We can find the slope (or the exponent) e by finding the distance between the two points in both the X and Y directions in the log-log plot and divide the two values.

$$\begin{aligned} e &= \frac{\ln aN^{\frac{1}{x}} - \ln 1}{\ln aN^{\frac{1}{x}}\frac{N(N-1)}{2} - \ln M} \approx \frac{\ln aN^{\frac{1}{x}}}{\ln aN^{\frac{1}{x}}\frac{N^2}{2} - \ln M} \\ &= \frac{\frac{1}{x}\ln N + \ln a}{(1+\frac{1}{x})\ln N + \ln\frac{a}{d}} \approx \frac{1}{1+x} \end{aligned}$$

For $x = 1$, $e \approx 0.5$, which is a good approximation for the ring. For $x = 2$, $e \approx \frac{1}{3}$, which is a good approximation for short-fiber topologies. Clearly, x, the dimension of a topology, largely determines the power-law exponent. As mentioned in Sec. IV-A, the power-law exponents for random topologies decrease for larger networks. This can be explained by the increase in the effective dimension of a random topology because of its large expansion factor.

B. Fiber and switching cost tradeoff

Now, we make use of these tradeoff lines to design cost-effective networks. The tradeoff line exhibits a knee on a linear graph, and there is a point on it where the total switching cost is minimized, as determined by the cost function. This is an optimal point given a particular topology. However, for a set of nodes, there are many possible topologies, depending on the physical connectivity, i.e., which fiber links are established. The position of the tradeoff line depends on the actual physical topology. To be precise, the position of the tradeoff line is determined by $\bar{H}_0$ (and N and M), because the coordinates of the end points on the $\bar{H}$ vs. S graph are $(M, \bar{H}_0)$ and $(\bar{H}_0 \frac{N(N-1)}{2}, 1)$. In Fig. 14, we show tradeoff lines corresponding to different amounts of fiber used for a 50-node network. When $\bar{H}_0$ decreases (corresponding to higher physical connectivity, or larger M), the tradeoff line moves in the direction of the origin. In other words, the switching requirements decrease (and thus the optimal total switching cost decreases) as the physical topology becomes more highly connected. Thus, we find not only a tradeoff between electronic and optical switching, but also a tradeoff between the switching cost and the fiber cost.

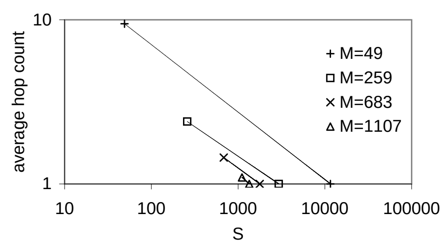

Fig. 14. Tradeoff lines for topologies with different amount of fiber, $N = 50$.

Given a set of node locations and a fiber length budget, we should design the fiber topology to minimize $\bar{H}_0$, which in turn minimizes the total switching cost. We use the following algorithm. First, we connect all nodes using a minimal spanning tree. Then, as long as the fiber length budget is not exceeded, we add one fiber at a time to enhance the connectivity. Let h_{ij} denote the number of physical hops and d_{ij} denote the fiber distance between i and j in the current fiber topology. When we add a fiber, we choose the pair of nodes with the highest $\frac{h_{ij}}{d_{ij}}$, thereby making the most effective use of the fiber. The topologies shown in Fig. 14 have been designed using this algorithm.

The total cost of switching includes the cost of optical switching, the cost of electronic switching and the cost of the interfaces. Let C_o be the cost of optically switching one lightpath at a node, let C_e be the cost of electronically switching one unit of traffic at a node, and let C_i be the interface cost for the transceivers required to terminate one lightpath. Then the total switching cost C_s could be expressed as:

$$C_s = C_o(S + L) + C_e \frac{N(N-1)}{2}(\bar{H} + 1)T + C_i L \quad (5)$$

To illustrate a network design scenario, we consider a network with 50 nodes. The above algorithm is used to design the fiber topology, for varying fiber distance budget. For each topology, we solve equation (5) to achieve the optimal switching cost. We set the following parameters for this scenario (though any weights may be used): $C_o = 1$, $C_e = 10$, $C_i = 2$ and $T = 0.1$. The resulting optimal S, L, H (where $H = \frac{N(N-1)}{2}\bar{H}$), and total switching cost C_s are shown in Fig. 15.

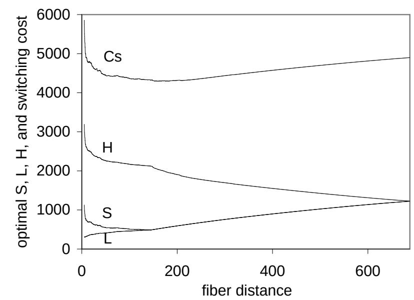

Fig. 15. S, L, H and C_s in the optimal solution for various topologies in a 50-node network.

At the beginning, as we increase the fiber length, the costs of both optical and electronic switching (S and $\bar{H}$) decrease. The decrease in $\bar{H}$ is a direct result of the increase in L. In this region, the total switching cost C_s decreases, at the expense of the fiber distance cost. The optimal solutions in these cases use a combination of electronic and optical switching.

As S continues to decrease and as L continues to increase, at some point (when the fiber distance is around 150, in this scenario), S becomes the same as L. The number of links becomes so large that even setting up a single lightpath on each fiber link (the electronic switching extreme) incurs high optical and interface cost. Beyond this point, there is no more tradeoff between switching cost and fiber cost—adding fiber links pushes up the fiber cost but does not help decrease the switching cost. In the figure, we show what happens if the fiber distance continues to increase: the optimal operating point is the electronic switching extreme, the interface and optical costs dominate, and the total switching cost goes up. The point at which adding more fibers does not help lower the total switching cost is dependent on the relative cost ratios. If C_o and C_i are higher, the transition will happen when the total fiber length is smaller.

In summary, we have shown how the optimal network design can be found readily, given only the cost weights and the node locations, and making use of the power-law tradeoff relationships. The solution yields the optimal fiber distance and specifies the optimal amount of switching equipment—routers, OXCs, and transceivers—at every node.

VI. Limited Number of Switching Locations

Thus far, we have assumed that switching may be performed at any node where it is needed. However, in some implementations, the local-interface switching hardware may be separate from the electronic switches handling transit traffic, e.g., implemented using multiplexers and demultiplexers rather than more complex switches. This raises a question—whether transit traffic switching is needed at all N nodes. Because there is a fixed cost associated with installing a switch at a node (e.g., the cost of a router chassis), regardless of switching load, it is desirable to minimize the number of switching locations (routers) R. In this section, we show that routers for switching transit traffic need to be installed at a small percentage of nodes, and the average hop count does not increase much when packets are forced to route through these fixed switching locations.

Let us first consider the ring and assume all lightpaths up to t hops long are established. To minimize R, we just need to add switching capability to every t nodes. This allows any node to reach a switch node, and each switch node to reach at least one other switch node. Therefore, $R = \frac{N}{t}$. As we establish more and more lightpaths (t increases), we need to install fewer and fewer routers.

Let us now consider a two-dimensional topology: the torus. Because all lightpaths up to t hops long are established, we can add switching capability to every t nodes in a row and we only need to do this for every t rows. Since there are $2k+1$ nodes in a row and there are $2k+1$ rows, we have

$$R = \frac{2k+1}{t}\frac{2k+1}{t} = \left(\frac{2k+1}{t}\right)^2$$

Compared to the rings, R reduces faster in the torus topology as t increases, because R is proportional to t^{-2} instead of t^{-1}. Because of the similarity, we expect the fast reduction in R to be true for all short-fiber topologies as well.

For both rings and tori, when switching is forced to go through these fixed locations, the average hop count should increase at most by 1. This is because a packet will take at most one logical hop to get to one of the routers (because all lightpaths up to t hops long are established), and from there, it will hop through t-hops-long lightpaths before it reaches the destination, just like it would do without the switching location restriction. Essentially, the t-hops-long lightpaths between routers become the backbone of the network. Without the switching location restriction, packets will still want to use t-hops-long lightpaths as much as possible to reduce the hop count. The switching location restriction only forces the packets to go on a limited number of these t-hops-long lightpaths and may also force some packets to go through one more hop in order to get onto the backbone.

For non-regular topologies, we use the following algorithm to determine where to place routers. Given W (or S), we first use a modified version of the SLF algorithm [4] to establish the logical topology. We then pick the node that has the highest node degree in the logical topology as it has the most neighbors, place a router there, and then mark all nodes that are one logical hop away (the neighbors) as *covered*. Next, among all the covered nodes, we pick the one that reaches the most number of un-covered nodes in one logical hop. We place a router at that node and mark all of its logical neighbors as covered. We repeat the last step of picking the node with the most un-covered neighbors until all nodes are covered.

To illustrate the effect of limiting the number of routers, we consider an arbitrary short-fiber topology with $N = 200$ nodes and an average node degree of 4. In Fig. 16, we show the number of routers needed and the corresponding increase in hop count if switching locations are limited. First, $\bar{H}$ is almost the same if many wavelengths are available. When W is limited, $\bar{H}$ increases because the logical topology becomes less and less well-connected, such that many flows take one extra hop. For all the cases we studied, $\bar{H}$ always increases by less than 1. Considering the number of routers R, when W is large, few routers are needed; R is 1 or 2 for a large range of W. Only when W is very small, R starts to increase. With a factor of 20 reduction in W compared to the value of W at the optical switching extreme, for the topologies we studied with N ranging from 50 to 200, R ranges from 6 to 16. This is a very small number of routers compared to the size of the network that we are studying.

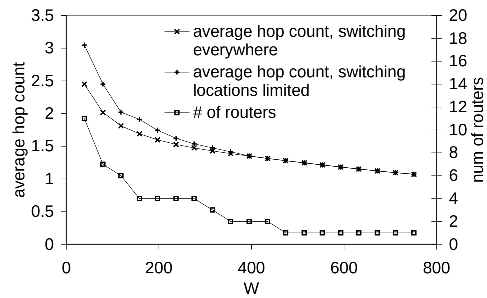

Fig. 16. Minimum number of routers needed and the resulting increase in the average hop count for a short-fiber topology with $N = 200$ and an average node degree of 4.

VII. Tradeoffs under generalized traffic

A. When is traffic relatively small?

So far we have assumed that the amount of traffic T for each flow is small compared to the bandwidth of the lightpaths and, therefore, all traffic can be routed along the shortest path. In this section, we determine when this holds true, and in Sec. VII-B we will show that $\bar{H}$ will increase by at most a small constant factor if rerouting onto alternate paths becomes necessary.

We are interested in the value of T such that all traffic can be routed along the shortest path, i.e., the maximum magnitude of the traffic demands such that the lightpath capacity is not an issue. In this range, we know that the power-law relationship holds.

Let us first consider rings. Let us assume that we can establish all lightpaths that are up to t physical hops long. All flows that are more than t physical hops long will try to use these t-hops-long lightpaths as much as possible in order to minimize the logical hop count. As a result, these t-hops-long lightpaths are the most overloaded; thus, we are interested in their traffic load.

For flows that are i physical hops long, each one of them will use $\lfloor \frac{i}{t} \rfloor$ separate t-hops-long lightpaths. Due to symmetry in the LLD, flows that are $N/2+1-i$ physical hops long take $\lfloor \frac{N/2+1-i}{t} \rfloor$ separate t-hops-long lightpaths. Ignoring the flooring operators, the average of the two is $\frac{N/2+1}{2t}$, which holds for all i. Summing over all $\frac{N(N-1)}{2}$ flows, then dividing by the number of t-hops-long lightpaths (assuming traffic is evenly spread among them), we find the traffic carried on each of the t-hops-long lightpaths.

$$T_t = \frac{1}{N}\frac{N(N-1)}{2}\frac{N/2+1}{2t}T \approx \frac{N^2}{8t}T$$

When the t-hops-long lightpaths are fully loaded, $T_t = 1$. Therefore, $T \leq \frac{8t}{N^2}$ will guarantee that all demands can go through the shortest path routes. The traffic limit is proportional to t because as we increase t, we increase the number of established lightpaths proportionally. In addition, the traffic limit is proportional to N^{-2} because there are only two fibers in any cross-section cut in the ring topology, but there are on the order of N^2 flows crossing the cut.

We can similarly derive the condition for the torus topology. We will just look at the case of $t \leq k$. For flows that are i physical hops long and for flows that are $2k+1-i$ physical hops long, on average, ignoring the flooring operators, each one will use $\frac{2k+1}{2t}$ separate t-hops-long lightpaths. This is true for all i. As we did for the ring, we find the traffic carried on each of the t-hops-long lightpaths.

$$T_t = \frac{1}{2Nt}\frac{N(N-1)}{2}\frac{2k+1}{2t}T \approx \frac{k^3}{t^2}T$$

Therefore, when $T \leq \frac{t^2}{k^3} \approx \frac{8t^2}{N^{\frac{3}{2}}}$, all traffic can go through the shortest path. The traffic limit is proportional to t^2 because we incrementally establish more lightpaths as we increase t. In addition, the traffic limit is proportional to $N^{-\frac{3}{2}}$ because there are $O(N^{\frac{1}{2}})$ fibers in a cross-section cut in a two-dimensional topology, such as the torus, and there are $O(N^2)$ flows crossing the cut.

The limit on T, below which traffic can be considered small compared to the lightpath capacity, increases as t increases. Therefore, we can always establish more lightpaths to ensure shortest path routing, which results in the minimum hop count, is used.

B. What happens when traffic is not small?

If T is not relatively small, the results derived in Sec. III and Sec. IV no longer hold. The average hop count will inevitably increase because traffic has to be diverged to non-shortest paths to even out the load. In this section, we derive the average hop count under the extreme condition where the traffic T is scaled up to as much as the capacity can support.

Let us first determine the maximum amount of traffic T that can be carried. Let W_a denote the number of wavelengths needed when $T = 1$, i.e., when a direct lightpath needs to be established between each pair of nodes. In rings, $W_a = 1+2+\ldots+\frac{N}{2} = \frac{N^2}{8}+\frac{N}{4} \approx \frac{N^2}{8}$. If W is a fraction of W_a, i.e., $W = \frac{W_a}{f}$, the capacity (the bandwidth distance product) is reduced by a factor of f. In the best case, we should be able to route $T = \frac{1}{f}$ traffic per flow. We can determine $f = \frac{W_a}{W} = \frac{N^2/8}{t^2/2} = (\frac{N}{2t})^2$.

To derive the average hop count, we define *external traffic* and *internal traffic*. External traffic is the total amount of traffic injected into the network from outside, i.e., from the metro area network. In our case, it is $T_{ext} = \frac{N(N-1)}{2}T \approx \frac{N^2}{2}T$. The internal traffic T_{int} refers to the total traffic carried by all established lightpaths, i.e., as seen from the routers' perspective. The ratio of the two defines the average hop count, i.e., $\bar{H} = T_{int}/T_{ext}$.

When $T = \frac{1}{f}$, all established lightpaths should be carrying traffic at their full capacity in order to accommodate the increased load. Since each lightpath has capacity of 1, the total internal traffic should be just the total number of established lightpaths. For rings, $T_{int} = Nt$. Therefore, the average hop count is

$$\bar{H} = \frac{T_{int}}{T_{ext}} = \frac{Nt}{\frac{N^2}{2}T} = \frac{2tf}{N} = \frac{2t}{N}\left(\frac{N}{2t}\right)^2 = \frac{N}{2t}$$

Compared to equation (1), $\bar{H}$ doubles. This can be intuitively explained as follows. When traffic increases, we cannot use the t-hops-long lightpaths because they are overloaded. However, we can first send traffic to an intermediate node by using a lightpath that is $j < t$ hops long, then send the traffic on a lightpath that is $t-j$ hops long in order to reach the node that is t hops away. Since all $t-1$ nodes in between can be used as relay nodes, we can spread out the traffic evenly onto all lightpaths. By using only one intermediate node, we only double the average hop count.

Similarly, we can derive $\bar{H}$ for tori when $T = \frac{1}{f}$. Again, let us only focus on the case of $t \leq k$. In tori, $W_a \approx k^3$ (details in [4]). We can then derive $f = \frac{W_a}{W} = 3\frac{k^3}{t^3}$. Since the total internal traffic is $T_{int} = \frac{4N}{2}\sum_{i=1}^{t} i \approx Nt^2$, the average hop count is

$$\bar{H} = \frac{Nt^2}{\frac{N^2}{2}T} = \frac{2t^2}{N}f = \frac{2t^2}{N}\frac{3k^3}{t^3} \approx \frac{2t^2}{4k^2}\frac{3k^3}{t^3} = \frac{3}{2}\frac{k}{t}.$$

In the derivation above, we used the fact that $N \approx 4k^2$.

Compared to equation (4), the average hop count increases by a factor of $\frac{3}{2}$, less than the increase in rings. This is because there are more longer lightpaths. When traffic is spread over all lightpaths, the longer lightpaths take more traffic. As a result, the average hop count does not increase as much.

We should point out that the factor of 2 increase in rings and the factor of $\frac{3}{2}$ increase in tori are the worst case when the traffic is at its highest. If the traffic is less than the maximum possible, the increase in $\bar{H}$ will be less.

For arbitrary physical topologies, we can also determine the maximum amount of traffic that can be handled in the logical topology and the corresponding increase in $\bar{H}$. This problem turns out to be the well known *minimum congestion problem* or the *maximum concurrent flow problem* in the algorithm community. The problem can be formulated as a Linear Program or can be solved by efficient algorithms such as the flow deviation method [11] [12].

In Fig. 17, we show the increase in $\bar{H}$ when the traffic is at its highest for an arbitrary short-fiber topology with $N = 100$ nodes and an average node degree of 4. The increase is large only when W is small, where the logical topology is poorly connected and the logical path lengths are large. When packets are rerouted away from the shortest path, the increase in path length will be larger. In an arbitrary short-fiber topology, we have found that the maximum amount of traffic that can be carried could be very low, e.g., around 30% or 40% of $\frac{1}{f}$.

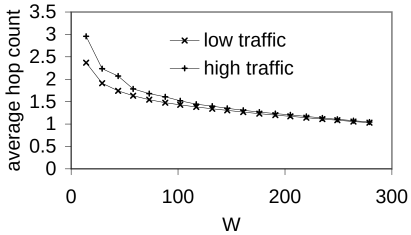

Fig. 17. Average hop count when T is low and when T is high for a short-fiber topology with $N = 100$ and an average node degree of 4.

C. Non-uniform traffic demands

In Sec. III and Sec. IV, we assumed that the traffic demands are uniform. If the traffic demands are not uniform, the analytical methods cannot be applied. However, it is straightforward to extend the simulation method to study non-uniform traffic. Instead of first picking the shortest lightpaths, we pick the lightpaths with the smallest $\frac{h_{ij}}{T_{ij}}$, where h_{ij} is the physical hop distance and T_{ij} is the amount of traffic between nodes i and j. Under this extended algorithm, short lightpaths that carry more direct traffic are connected first.

We first consider *randomized* traffic demands, where the traffic demand T_{ij} between node i and j is a random variable uniformly chosen from a range $(0, \rho)$ that is common for all node pairs. In Fig. 18, we show the tradeoff between $\bar{H}$ and S for two different arbitrary short-fiber topologies under uniform and randomized traffic patterns. The figure shows that the tradeoff under randomized traffic is very similar to that under uniform traffic. This is because the uniform distribution of randomized demands does not significantly alter the order that the lightpaths are chosen and the weight of each demand. Note that the size of the range (the value of ρ) does not make any difference, since it is the relative weight of the demands that determines the order the lightpaths are chosen, and the relative weight that affects $\bar{H}$.

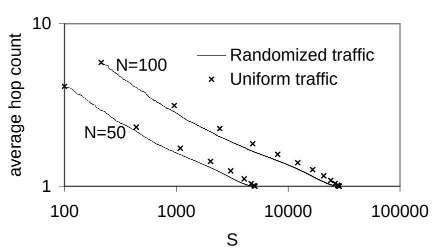

Fig. 18. Average hop count under uniform and randomized traffic demands for arbitrary short-fiber topologies with a fiber-to-node ratio of 2 and with $N = 50$ and $N = 100$.

For other traffic patterns, namely ones that grossly skew the distribution of demand magnitudes, the tradeoff could differ. For example, we considered a demand pattern where half of the flows have a magnitude that is 100 times that of the other half. The result is a tradeoff showing two separate regions. Toward the electronic switching extreme, the tradeoff is determined by the LLD for the node pairs with larger T_{ij} because lightpaths are first placed among these node pairs; the heavier weight of the large demands affects $\bar{H}$ significantly. Toward the optical switching extreme, the tradeoff is determined by the LLD for the remaining node pairs; as additional optical resources are offered, the reduction in $\bar{H}$ is minimal.

VIII. CONCLUSION

We have shown that the tradeoff between optical switching (W, S and L) and electronic switching ($\bar{H}$) can be described by a power-law relationship. We have quantified the power-law exponents for many classes of topology and have shown how this exponent may vary for very large networks.

We have also identified a simple, but important, design principle. Namely, when designing a network with a combination of optical and electronic switching, shorter lightpaths should be favored. This is because shorter lightpaths use fewer optical resources, and, to a certain degree, a short lightpath is equivalent to a long lightpath from the point of view of supporting traffic. Network designers can benefit greatly from paying attention to the lightpath length distribution.

Knowing that the tradeoff follows a power-law, a network designer can quickly derive the tradeoff curve by finding two coordinates—one at the optical switching extreme and one at the electronic switching extreme. We have shown how this procedure can be applied to evaluate the tradeoff between fiber and switching equipment. Finally, we have shown how to employ these insights and understanding of the tradeoffs to design a fiber network that achieves the minimal total cost, which includes the fiber cost, as well as the electronic switching, optical switching, and transceiver costs.

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